Momentum and Impulse (Key)

1) Under what circumstances could an object initially at rest be struck and move at a greater speed after collision than the incoming object?
2) A 180-kg bumper car carrying a 70-kg driver has a constant velocity of 3.0 m/s [E]. Calculate the momentum of the car-driver system. [7.5 x 10^2 kgm/s]

Given

$m_c = 180$ kg $\quad m_d = 70$ kg $\quad \vec{v} = 3.0$ m/s [E]

The driver and bumper car are a system because they move together as a unit. Find the total mass of the system.

$$\begin{aligned} m_T &= m_c + m_d \\ &= 180 \text{ kg} + 70 \text{ kg} \\ &= 250 \text{ kg} \end{aligned}$$

$$\begin{aligned} p &= m_T v \\ &= (250 \text{ kg})(3.0 \text{ m/s}) \\ &= 7.5 \times 10^2 \text{ kg·m/s} \end{aligned}$$

The momentum of the car-driver system is 7.5×10^2 kg·m/s [E].

3) A 65-kg girl is driving a 535-kg snowmobile at a constant velocity of 11.5 m/s [60.0° N of E]. Calculate the momentum of the girl-snowmobile system. [6.90 x 10^3 kgm/s [E60° N]]

Given

$m_g = 65$ kg $\quad \vec{v} = 11.5$ m/s [60.0° N of E]

$m_s = 535$ kg

The girl and snowmobile are a system because they move together as a unit. Find the total mass of the system.

$$\begin{aligned} m_T &= m_g + m_s \\ &= 65 \text{ kg} + 535 \text{ kg} \\ &= 600 \text{ kg} \end{aligned}$$

The momentum of the system is in the direction of the velocity of the system. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$$\begin{aligned} p &= m_T v \\ &= (600 \text{ kg})(11.5 \text{ m/s}) \\ &= 6.9 \times 10^3 \text{ kg·m/s} \end{aligned}$$

Paraphrase

The momentum of the girl-snowmobile system is 6.9×10^3 kg·m/s [60.0° N of E].

4) The combined mass of a bobsled and two riders is 390 kg. The sled-rider system has a constant momentum of 4.68×10^3 kgm/s [W]. Calculate the velocity of the sled. [12.0 m/s [W]]

Given

$m_T = 390$ kg

$\vec{p} = 4.68 \times 10^3$ kg•m/s [W]

The momentum of the system is in the direction of the velocity of the system.
So use the scalar form of $\vec{p} = m\vec{v}$ to find the speed.

$$p = m_T v$$

$$v = \frac{p}{m_T}$$

$$= \frac{4.68 \times 10^3\ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}}}{390\ \cancel{\text{kg}}}$$

$$= 12.0 \text{ m/s}$$

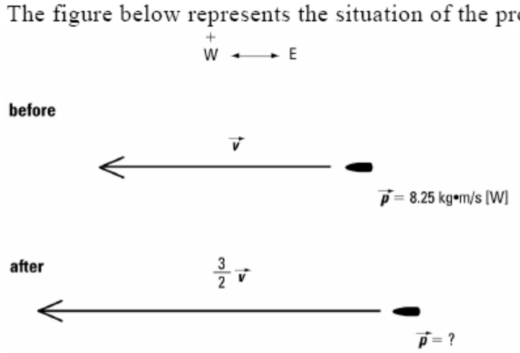
Paraphrase

The velocity of the sled is 12.0 m/s [W].

5) Many modern rifles use bullets that have less mass and reach higher speeds than bullets for older rifles, resulting in increased accuracy over longer distances. The momentum of a bullet is initially 8.25 kgm/s [W]. What is the momentum if the speed of the bullet increases by a factor of $\frac{3}{2}$ and its mass decreases by a factor of $\frac{3}{4}$? [9.28 kgm/s [W]]

From the equation $\vec{p} = m\vec{v}$, $p \propto m$ and $p \propto v$.

The figure below represents the situation of the problem.



$p \propto \frac{3}{4} m$ and $p \propto \frac{3}{2} v$

Calculate the factor change of p.

$$\frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$$

Calculate p.

$$\frac{9}{8} p = \frac{9}{8} \times (8.25 \text{ kg•m/s})$$

$$= 9.28 \text{ kg•m/s}$$

The new momentum will be 9.28 kg•m/s [W].

6) Explain why momentum is a vector quantity.

7) A hockey puck has a momentum of 3.8 kgm/s [E]. If its speed is 24 m/s, what is the mass of the puck? [0.16 kg]

Given

$\vec{p}$ = 3.8 kg•m/s [E]

v = 24 m/s

The momentum of the puck is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the mass.

$$p = mv$$

$$m = \frac{p}{v}$$

$$= \frac{3.8\ \text{kg}\cdot\cancel{\frac{\text{m}}{\text{s}}}}{24\ \cancel{\frac{\text{m}}{\text{s}}}}$$

$$= 0.16\ \text{kg}$$

8) A loaded transport truck with a mass of 38 000 kg is travelling at 1.20 m/s [W]. What will be the velocity of a 1400-kg car if it has the same momentum? [32.6 m/s [W]]

Given

m_t = 38 000 kg

$\vec{v}_t$ = 1.20 m/s [W]

m_c = 1400 kg

The momentum of each vehicle is directed west. So use the scalar form of $\vec{p} = m\vec{v}$ to get an expression for the magnitude of the momentum of each vehicle.

$p_t = m_t v_t$ and $p_c = m_c v_c$

Since the momentum of each vehicle is the same, set both equations equal to each other.

$$m_t v_t = m_c v_c$$

Solve to find the speed of the car.

$$v_c = \left(\frac{m_t}{m_c}\right) v_t$$

$$= \left(\frac{38\ 000\ \cancel{\text{kg}}}{1400\ \cancel{\text{kg}}}\right)(1.20\ \text{m/s})$$

$$= 32.6\ \text{m/s}$$

Paraphrase and Verify

The velocity of the car would have to be 32.6 m/s [W] in order to have the same momentum as the truck. Since the car is about 30 times less massive than the truck, you would expect the car to be travelling about 30 times faster than the truck. So the calculated answer is reasonable.

9) To improve the safety of motorists, modern cars are built so the front end crumples upon impact. A 1200-kg car is travelling at a constant velocity of 8.0 m/s [E]. It hits an immovable wall and comes to a complete stop in 0.25 s.

i) Calculate the impulse provided to the car. [9.6×10^3 Ns [W]]

$m = 1200$ kg $\qquad \vec{v}_i = 8.0$ m/s [E]

(a) and (b) $\Delta t = 0.25$ s

$$\begin{aligned}
\vec{F}_{\text{net}_{\text{ave}}}\Delta t &= m\Delta\vec{v} \\
&= m(\vec{v}_f - \vec{v}_i) \\
&= (1200 \text{ kg})[0 - (+8.0 \text{ m/s})] \\
&= (1200 \text{ kg})(-8.0 \text{ m/s}) \\
&= -9.6 \times 10^3 \text{ kg·m/s} \\
\text{impulse} &= 9.6 \times 10^3 \text{ N·s [W]}
\end{aligned}$$

ii) What is the average net force exerted on the car? [3.8×10^4 N [W]]

$$\begin{aligned}
\vec{F}_{\text{net}_{\text{ave}}}\Delta t &= -9.6 \times 10^3 \text{ N·s} \\
\vec{F}_{\text{net}_{\text{ave}}} &= \frac{-9.6 \times 10^3 \text{ N·s}}{\Delta t}
\end{aligned}$$

$$\begin{aligned}
\vec{F}_{\text{net}_{\text{ave}}} &= \frac{-9.6 \times 10^3 \text{ N·s}}{0.25 \text{ s}} \\
&= -3.8 \times 10^4 \frac{\text{kg·m}}{\text{s}^2} \\
&= -3.8 \times 10^4 \text{ N} \\
\vec{F}_{\text{net}_{\text{ave}}} &= 3.8 \times 10^4 \text{ N [W]}
\end{aligned}$$

10) For the same impulse, what would be the average net force exerted on the car if it had a rigid bumper and frame that stopped the car in 0.040 s? [2.4×10^5 N [W]]

$$\begin{aligned}
\vec{F}_{\text{net}_{\text{ave}}} &= \frac{-9.6 \times 10^3 \text{ N·s}}{0.040 \text{ s}} \\
&= -2.4 \times 10^5 \frac{\text{kg·m}}{\text{s}^2} \\
&= -2.4 \times 10^5 \text{ N} \\
\vec{F}_{\text{net}_{\text{ave}}} &= 2.4 \times 10^5 \text{ N [W]}
\end{aligned}$$

11) Two people push a car for 3.64 s with a combined net force of 200 N [S].

i) Calculate the impulse provided to the car. [728 Ns [S]]

$\Delta t = 3.64$ s

$\vec{F}_{net} = 200$ N [S]

$$\begin{aligned} \text{impulse} &= \vec{F}_{net}\,\Delta t \\ &= (+200\ \text{N})(3.64\ \text{s}) \\ &= +728\ \text{N}\bullet\text{s} \\ \text{impulse} &= 728\ \text{N}\bullet\text{s [S]} \end{aligned}$$

ii) If the car has a mass of 1100 kg, what will be its change in velocity? [0.662 m/s [S]]

impulse = 728 N•s [S] from part (a)

m = 1100 kg

$$\begin{aligned} +728\ \text{N}\bullet\text{s} &= m\Delta\vec{v} \\ \Delta\vec{v} &= \frac{+728\ \text{N}\bullet\text{s}}{m} \\ &= \frac{+728\ \text{N}\bullet\text{s}}{1100\ \text{kg}} \\ &= \frac{+728\left(\cancel{\text{kg}}\bullet\frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}})}{1100\ \cancel{\text{kg}}} \\ &= +0.662\ \text{m/s} \\ \Delta\vec{v} &= 0.662\ \text{m/s [S]} \end{aligned}$$

12) A dog team pulls a 400-kg sled that has begun to slide backward. In 4.20 s, the velocity of the sled changes from 0.200 m/s [backward] to 1.80 m/s [forward]. Calculate the average net force the dog team exerts on the sled. [190 N [Forward]]

$m = 400$ kg

$\vec{v}_i = 0.200$ m/s [backward]

$\Delta t = 4.20$ s

$\vec{v}_f = 1.80$ m/s [forward]

Use the equation of impulse to calculate the average net force that the dog team exerts on the sled.

$$\begin{aligned} \vec{F}_{net_{ave}}\,\Delta t &= m\Delta\vec{v} \\ \vec{F}_{net_{ave}} &= \frac{m\Delta\vec{v}}{\Delta t} \\ &= \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t} \\ &= \frac{(400\ \text{kg})\{+1.80\ \text{m/s} - (-0.200\ \text{m/s})\}}{4.20\ \text{s}} \\ &= +190\ \text{kg}\bullet\text{m/s}^2 \\ \vec{F}_{net_{ave}} &= 190\ \text{N [forward]} \end{aligned}$$

13) How are impulse and momentum related?

Impulse is equivalent to the change in momentum of an object.

14) Even though your mass is much greater than that of a curling stone, it is dangerous for a moving stone to hit your feet. Explain why.

Curling stones used in championship games have a maximum mass of 19.96 kg. When a curling stone is moving, it has a lot of inertia because its mass is fairly large. Since a curling stone is a rigid object, the time interval during a collision would be very short. If a stone were to hit your feet, the impact would be very painful since the bones and soft tissue in your feet would have to cause the stone to stop moving in a short time interval.

15) Experienced curlers know how to safely stop a moving stone. What do they do and why?

There are two ways to safely stop a moving curling stone:

- Stop sweeping the ice in front of the stone. Sweeping causes the stone to travel farther because the force of friction acting on the stone decreases. By not sweeping, the friction on the stone increases, which in turn causes the stone to stop sooner.
- Use a rigid object such as the curling broom to slow down the stone. By causing the stone to collide with the broom, the speed of the stone can be decreased more gradually, so the interaction time is increased.

16) What will be the magnitude of the impulse generated by a slapshot when an average net force of magnitude 520 N is applied to a puck for 0.012 s? [6.24 Ns [forward]]

$F_{\text{net}_{\text{ave}}} = 520\text{ N}$

$\Delta t = 0.012\text{ s}$

Use the equation of impulse to calculate the impulse provided to the puck.

$$\begin{aligned}\text{impulse} &= \bar{F}_{\text{net}_{\text{ave}}}\Delta t\\ &= (+520\text{ N})(0.012\text{ s})\\ &= +6.2\text{ N}\cdot\text{s}\\ \text{impulse} &= 6.2\text{ N}\cdot\text{s [forward]}\end{aligned}$$